4-4 Using Corresponding Parts of Congruent Triangles

Common Core State Standards

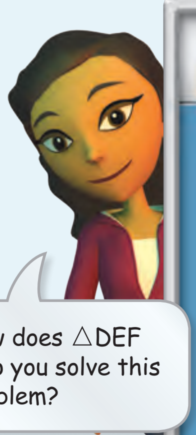
G-SRT.B.5 Use congruence . . . criteria for triangles to solve problems and prove relationships in geometric figures. **Also G-CO.D.12**

Mathematics Florida Standards

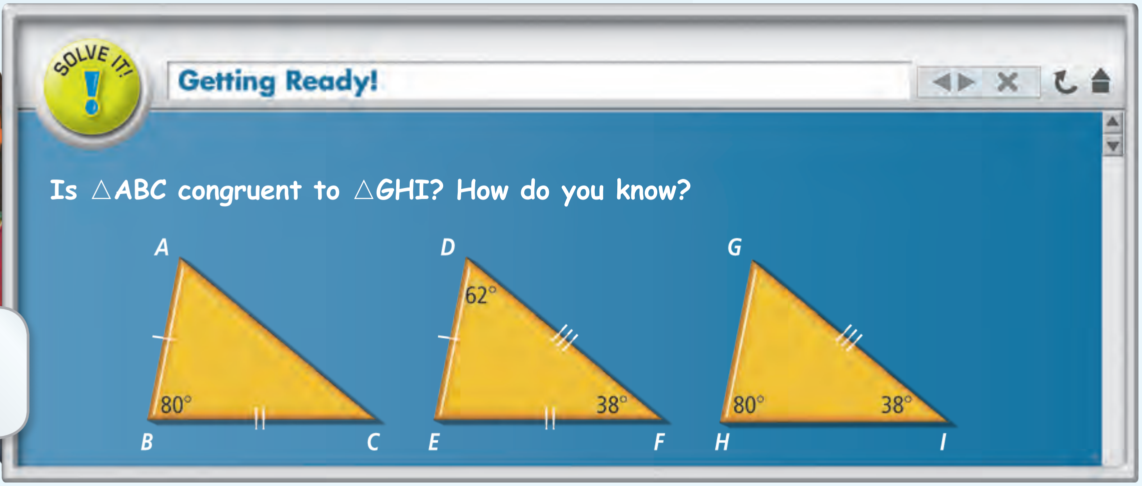
MAFS.912.G-SRT.2.5 Use congruence criteria for triangles to solve problems and to prove relationships in geometric figures. **Also MAFS.912.G-SRT.4.12**

MP 1, MP 3

Objective To use triangle congruence and corresponding parts of congruent triangles to prove that parts of two triangles are congruent


How does △DEF help you solve this problem?

MATHEMATICAL PRACTICES



Is △ABC congruent to △GHI? How do you know?

With SSS, SAS, ASA, and AAS, you know how to use three congruent parts of two triangles to show that the triangles are congruent. Once you know that two triangles are congruent, you can make conclusions about their other corresponding parts because, by definition, corresponding parts of congruent triangles are congruent.

Essential Understanding If you know two triangles are congruent, then you know that every pair of their corresponding parts is also congruent.

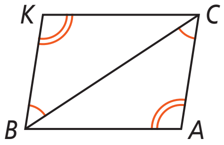
Problem 1 Proving Parts of Triangles Congruent

Think **Proof**

In the diagram, which congruent pair is not marked?
The third angles of both triangles are congruent. But there is no AAA congruence rule. So, find a congruent pair of sides.

Given: $\angle KBC \cong \angle ACB$, $\angle K \cong \angle A$

Prove: $\overline{KB} \cong \overline{AC}$



$\angle KBC \cong \angle ACB$
Given

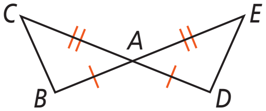
$\overline{BC} \cong \overline{BC}$
Reflexive Property of ≅

$\angle K \cong \angle A$
Given

$\triangle KBC \cong \triangle ACB$
AAS Theorem

$\overline{KB} \cong \overline{AC}$
Corresp. parts of ≅ △ are ≅.

Got It? **1. Given:** $\overline{BA} \cong \overline{DA}$, $\overline{CA} \cong \overline{EA}$
Prove: $\angle C \cong \angle E$



Proof Problem 2 Proving Triangle Parts Congruent to Measure Distance STEM

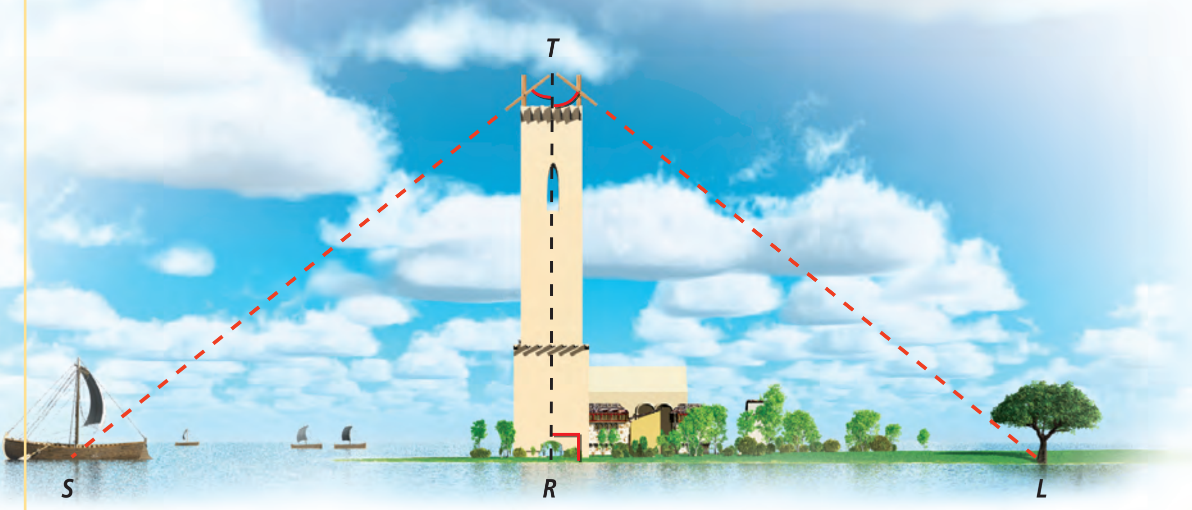
Plan

Which congruency rule can you use? You have information about two pairs of angles. *Guess-and-check* AAS and ASA.

Measurement Thales, a Greek philosopher, is said to have developed a method to measure the distance to a ship at sea. He made a compass by nailing two sticks together. Standing on top of a tower, he would hold one stick vertical and tilt the other until he could see the ship S along the line of the tilted stick. With this compass setting, he would find a landmark L on the shore along the line of the tilted stick. How far would the ship be from the base of the tower?

Given: $\angle TRS$ and $\angle TRL$ are right angles, $\angle RTS \cong \angle RTL$

Prove: $\overline{RS} \cong \overline{RL}$



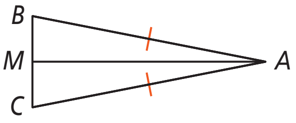
Statements	Reasons
1) $\angle RTS \cong \angle RTL$	1) Given
2) $\overline{TR} \cong \overline{TR}$	2) Reflexive Property of Congruence
3) $\angle TRS$ and $\angle TRL$ are right angles.	3) Given
4) $\angle TRS \cong \angle TRL$	4) All right angles are congruent.
5) $\triangle TRS \cong \triangle TRL$	5) ASA Postulate
6) $\overline{RS} \cong \overline{RL}$	6) Corresponding parts of $\cong$ $\triangle$s are $\cong$.

The distance between the ship and the base of the tower would be the same as the distance between the base of the tower and the landmark.

Got It? **2. a. Given:** $\overline{AB} \cong \overline{AC}$, M is the midpoint of $\overline{BC}$

Prove: $\angle AMB \cong \angle AMC$

b. Reasoning If the landmark were not at sea level, would the method in Problem 2 work? Explain.

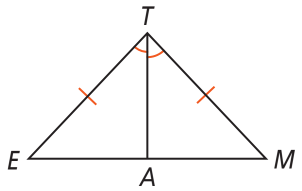


Lesson Check

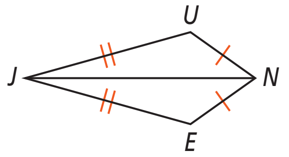
Do you know HOW?

Name the postulate or theorem that you can use to show the triangles are congruent. Then explain why the statement is true.

1. $\overline{EA} \cong \overline{MA}$



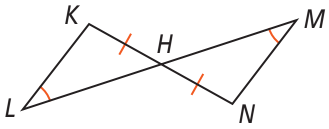
2. $\angle U \cong \angle E$



Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. Reasoning How does the fact that corresponding parts of congruent triangles are congruent relate to the definition of congruent triangles?

4. Error Analysis Find and correct the error(s) in the proof.



Given: $\overline{KH} \cong \overline{NH}$, $\angle L \cong \angle M$

Prove: H is the midpoint of $\overline{LM}$.

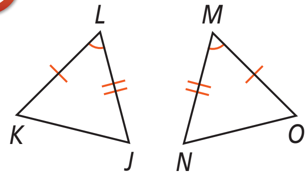
Proof: $\overline{KH} \cong \overline{NH}$ because it is given. $\angle L \cong \angle M$ because it is given. $\angle KHL \cong \angle NHM$ because vertical angles are congruent. So, $\triangle KHL \cong \triangle MHN$ by ASA Postulate. Since corresponding parts of congruent triangles are congruent, $\overline{LH} \cong \overline{MH}$. By the definition of midpoint, H is the midpoint of $\overline{LM}$.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

5. Developing Proof Tell why the two triangles are congruent. Give the congruence statement. Then list all the other corresponding parts of the triangles that are congruent.

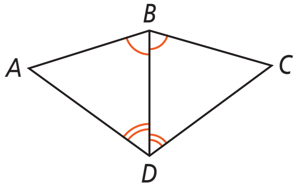


See Problem 1.

6. Given: $\angle ABD \cong \angle CBD$,
$\angle BDA \cong \angle BDC$

Proof

Prove: $\overline{AB} \cong \overline{CB}$

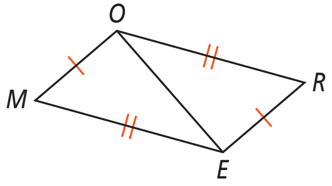


7. Given: $\overline{OM} \cong \overline{ER}$, $\overline{ME} \cong \overline{RO}$

Proof

Prove: $\angle M \cong \angle R$

See Problem 2.

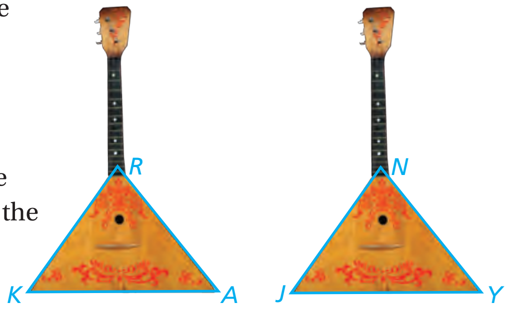


8. Developing Proof A balalaika is a stringed instrument. Prove that the bases of the balalaikas are congruent.

Given: $\overline{RA} \cong \overline{NY}$, $\angle KRA \cong \angle JNY$, $\angle KAR \cong \angle JYN$

Prove: $\overline{KA} \cong \overline{JY}$

Proof: It is given that two angles and the included side of one triangle are congruent to two angles and the included side of the other. So, **a.** $\underline{\ ?\ } \cong \triangle JNY$ by **b.** $\underline{\ ?\ }$. $\overline{KA} \cong \overline{JY}$ because **c.** $\underline{\ ?\ }$.

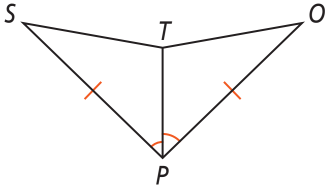


B Apply

9. Given: $\angle SPT \cong \angle OPT$,
$\overline{SP} \cong \overline{OP}$

Proof

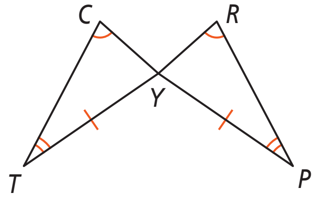
Prove: $\angle S \cong \angle O$



10. Given: $\overline{YT} \cong \overline{YP}$, $\angle C \cong \angle R$,
$\angle T \cong \angle P$

Proof

Prove: $\overline{CT} \cong \overline{RP}$

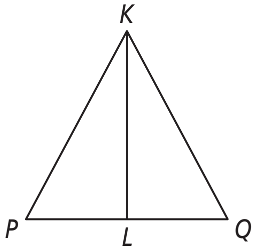


Reasoning Copy and mark the figure to show the given information. Explain how you would prove $\angle P \cong \angle Q$.

11. Given: $\overline{PK} \cong \overline{QK}$, $\overline{KL}$ bisects $\angle PKQ$

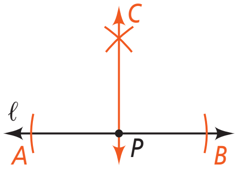
12. Given: $\overline{KL}$ is the perpendicular bisector of $\overline{PQ}$.

13. Given: $\overline{KL} \perp \overline{PQ}$, $\overline{KL}$ bisects $\angle PKQ$



14. Think About a Plan The construction of a line perpendicular to line ℓ through point P on line ℓ is shown. Explain why you can conclude that $\overleftrightarrow{CP}$ is perpendicular to ℓ.

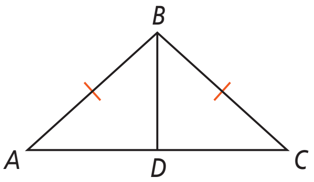
- How can you use congruent triangles to justify the construction?
- Which lengths or distances are equal by construction?



15. Given: $\overline{BA} \cong \overline{BC}$, $\overline{BD}$ bisects $\angle ABC$

Proof

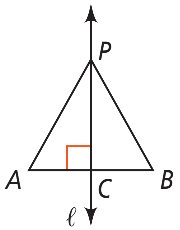
Prove: $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\overline{AC}$



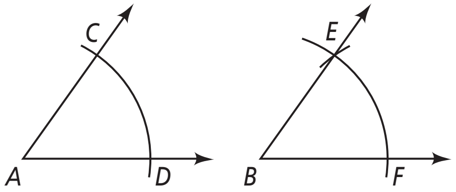
16. Given: $\ell \perp \overline{AB}$, ℓ bisects $\overline{AB}$ at C,
P is on ℓ

Proof

Prove: $PA = PB$



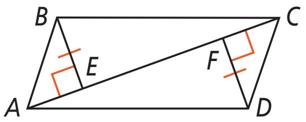
17. Constructions The construction of $\angle B$ congruent to given $\angle A$ is shown. $\overline{AD} \cong \overline{BF}$ because they are congruent radii. $\overline{DC} \cong \overline{FE}$ because both arcs have the same compass settings. Explain why you can conclude that $\angle A \cong \angle B$.



18. Given: $\overline{BE} \perp \overline{AC}$, $\overline{DF} \perp \overline{AC}$,
$\overline{BE} \cong \overline{DF}$, $\overline{AF} \cong \overline{CE}$

Proof

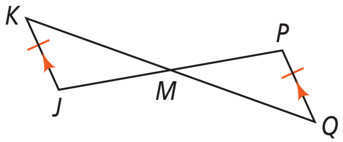
Prove: $\overline{AB} \cong \overline{CD}$



19. Given: $\overline{JK} \parallel \overline{QP}$, $\overline{JK} \cong \overline{PQ}$

Proof

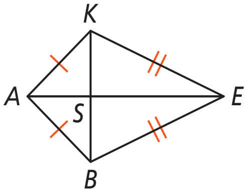
Prove: $\overline{KQ}$ bisects $\overline{JP}$.



20. Designs Rangoli is a colorful design pattern drawn outside houses in India, especially during festivals. Vina plans to use the pattern at the right as the base of her design. In this pattern, $\overline{RU}$, $\overline{SV}$, and $\overline{QT}$ bisect each other at O. $RS = 6$, $RU = 12$, $\overline{RU} \cong \overline{SV}$, $\overline{ST} \parallel \overline{RU}$, and $\overline{RS} \parallel \overline{QT}$. What is the perimeter of the hexagon?

Challenge

In the diagram at the right, $\overline{BA} \cong \overline{KA}$ and $\overline{BE} \cong \overline{KE}$.



21. Prove: S is the midpoint of $\overline{BK}$.

Proof

22. Prove: $\overline{BK} \perp \overline{AE}$

Proof

Apply What You've Learned

MATHEMATICAL PRACTICES

MP 7

Look back at the information on page 217 and at your work from the Apply What You've Learned sections in Lessons 4-1 and 4-3. Choose from the following words and names of figures to complete the sentences below.

congruent	corresponding	$\overline{AY}$	$\overline{AC}$
$\angle Y$	$\angle YAX$	$\angle X$	$\angle C$
$\triangle AYX$	$\triangle YAX$	$\overline{BC}$	$\overline{YX}$

In the Apply What You've Learned in Lesson 4-3, you proved that $\triangle BAC \cong$ **a.** ? .

Now, you can conclude that $\angle C \cong$ **b.** ? because **c.** ? parts of **d.** ? triangles are congruent.

Similarly, **e.** ? and $\overline{AX}$ are congruent corresponding sides. Another pair of congruent corresponding sides are **f.** ? and $\overline{YX}$.

Concept Byte

Use With Lesson 4-5

ACTIVITY

Paper-Folding Conjectures

Common Core State Standards

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (. . . paper folding . . .).

MP 3

Mathematics Florida Standards

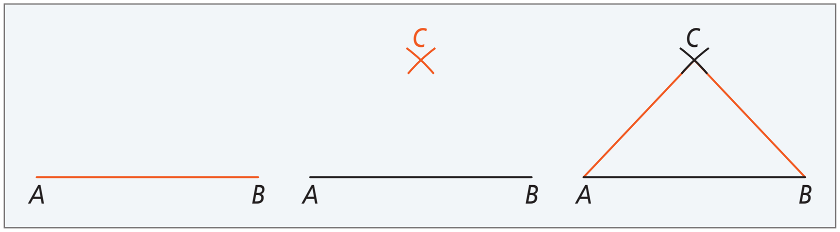
MAFS.912.G-CO.4.12 Make formal geometric constructions with a variety of tools and methods (. . . paper folding . . .).

MP 3

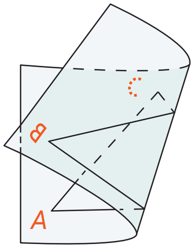
Isosceles triangles have two congruent sides. Folding one of the sides onto the other will suggest another important property of isosceles triangles.

Activity 1

Step 1 Construct an isosceles $\triangle ABC$ on tracing paper, with $\overline{AC} \cong \overline{BC}$.



Step 2 Fold the paper so the two congruent sides fit exactly one on top of the other. Crease the paper. Label the intersection of the fold line and $\overline{AB}$ as point D.



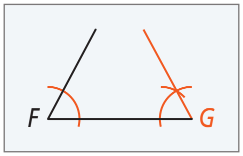
1. What do you notice about $\angle A$ and $\angle B$? Compare your results with others'. Make a conjecture about the angles opposite the congruent sides in an isosceles triangle.

2. a. Study the fold line $\overline{CD}$ and the base $\overline{AB}$. What type of angles are $\angle CDA$ and $\angle CDB$? How do $\overline{AD}$ and $\overline{BD}$ seem to be related?

b. Use your answers to part (a) to complete the conjecture:
The fold line $\overline{CD}$ is the $\underline{\ ?\ }$ of the base $\overline{AB}$ of isosceles $\triangle ABC$.

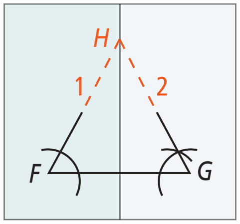
Activity 2

In Activity 1, you made a conjecture about angles opposite the congruent sides of a triangle. You can also fold paper to study whether the converse is true.



Step 1 On tracing paper, draw acute angle F and one side $\overline{FG}$. Construct $\angle G$ as shown, so that $\angle G \cong \angle F$.

Step 2 Fold the paper so $\angle F$ and $\angle G$ fit exactly one on top of the other.



3. Why do sides 1 and 2 meet at point H on the fold line? Make a conjecture about sides $\overline{FH}$ and $\overline{GH}$ opposite congruent angles in a triangle.

4. Write your conjectures from Questions 1 and 3 as a biconditional.